Supplemental Material – Generalized Streak Lines

Advected Tangent Curves: A General Scheme for Characteristic Curves of Flow Fields

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In section 3.2 of our paper, we introduce new types of characteristic curves that can be described using our general scheme of advected tangent curves. It turns out that generalized streak lines [WTS+07] can be described using this scheme. This is detailed in the following. Blue references such as (8) refer to formulas in the paper.

Generalized Streak Lines $\boxed{\mathbf{w} = \mathbf{v},\ \bar{\mathbf{f}} = \mathrm{FFF}}$

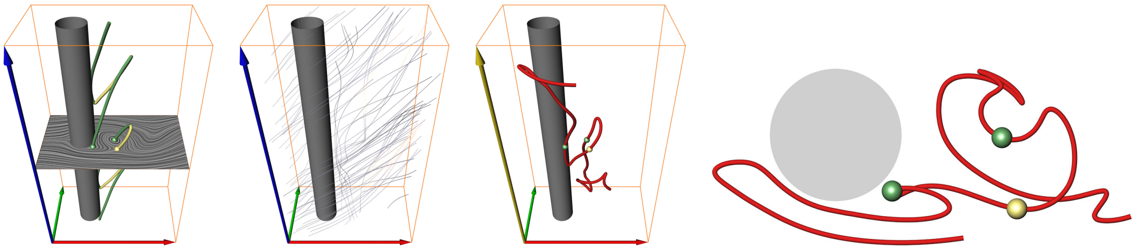
Wiebel et al. [WTS+07] introduced a generalization of streak lines where the spatial seeding location moves over time, i.e., particles are released from an arbitrary curve in space-time and their collection at a certain time step denotes the generalized streak line. Many different types of generalized streak lines are possible depending on the particular choice of seeding curves. Wiebel et al. used the paths of wall shear stress singularities as seeding curves in order to visualize boundary induced vortices.

We can formulate a tangent curve description for those types of generalized streak lines where the seeding curves are the tangent curves of some vector field. For example, the paths of the critical points of the flow itself can be described using Feature Flow Fields [TS03], i.e., the critical points of the time-dependent flow field $\mathbf{v}$ are line-type structures given as the tangent curves of

$$\bar{\mathbf{f}}_{2D}(\mathbf{x},t) = \begin{pmatrix} \det(\mathbf{v}_y, \mathbf{v}_t) \\ \det(\mathbf{v}_t, \mathbf{v}_x) \\ \det(\mathbf{v}_x, \mathbf{v}_y) \end{pmatrix} \quad \bar{\mathbf{f}}_{3D}(\mathbf{x},t) = \begin{pmatrix} +\det(\mathbf{v}_y, \mathbf{v}_z, \mathbf{v}_t) \\ -\det(\mathbf{v}_z, \mathbf{v}_t, \mathbf{v}_x) \\ +\det(\mathbf{v}_t, \mathbf{v}_x, \mathbf{v}_y) \\ -\det(\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z) \end{pmatrix}, \tag{1}$$

for 2D and 3D flows respectively, where $\mathbf{v}_x \ldots \mathbf{v}_t$ denote the partial derivatives of $\mathbf{v}$. More precisely, magnitude and direction of $\mathbf{v}$ remain constant along any tangent curve of these fields. Seeding a tangent curve of (1) at a critical point $\mathbf{v} = \mathbf{0}$ tracks this critical point over time. Seeding at a regular point $\mathbf{v} = \mathbf{b}$ tracks this particular vector $\mathbf{b}$ over time. Inserting (1) as the seeding field and $\bar{\mathbf{p}} = (\mathbf{v}, 1)$ as the advection field into (8) yields the tangent curve description of the corresponding generalized streak lines. This has been done in Figure 1 for a 2D time-dependent flow behind a cylinder. The feature flow field (Figure 1a) gets advected by the path lines of the flow (Figure 1b). In the resulting vector field, we seeded generalized streak lines from the critical points of the chosen time step (Figure 1c). Note that these lines coincide since the critical points are connected to each other in space-time via fold bifurcations. Also note that generalized streak lines may change their direction with respect to the τ-axis – this is not possible for the classic type of streak lines, see (23).

A more detailed analysis of generalized streak lines is left for future research.



(a) Seeding: critical points (green, yellow) tracked in space-time using FFF.

(b) Advection: Path lines in space-time.

(c) Three generalized streak lines in the domain $D \times \Upsilon$ (left) and their spatial projection (right). They have been seeded from three different critical points. The lines coincide since the critical points are connected to each other in space-time via fold bifurcations.

Figure 1: Generalized streak lines seeded from the locations of moving critical points in the 2D time-dependent flow behind a cylinder. The seeding field $\bar{\mathbf{f}}$ in (a) is shown only for the tracked critical points for better visual clarity, but the whole field has been used for the computations.

References

[TS03] H. Theisel and H.-P. Seidel. Feature flow fields. In *Data Visualization 2003. Proc. VisSym 03*, pages 141–148, 2003.

[WTS+07] Alexander Wiebel, Xavier Tricoche, Dominic Schneider, Heike Jaenicke, and Gerik Scheuermann. Generalized streak lines: Analysis and visualization of boundary induced vortices. *IEEE TVCG*, 13(6):1735–1742, 2007.